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On the Digital Time Paradigm

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Traditionally, people consider the computer as a tool, and hence the relation between science and computing is often treated from the utilitarian standpoint as using computers for calculating something of interest for a scientist, or performing some task of interest for an engineer. Even in mathematics, where general theories of computation have been developed since nearly a century ago [1], the computer is predominantly used for solving problems rather than formulating them [2, 3, 4]. However, computing can also be considered as one of the human activities and thus become a subject of scientific or philosophical study and open vistas that could revolutionize the scientific picture of the world. Leaving aside the attempts to apply the computer paradigm to the explanation of consciousness [5], or the extended computation models in logic [4, 6], I would stress the value of computing as a way of formalization, revealing the hidden regularities in the common procedures. Automation of human activities requires a thorough analysis and filtering, separating the regularities from fluctuations, which, sometimes, can lead to modification of the activities themselves. The experience of the development of the Information System of the ISTC has clearly demonstrated that. Now, applying the same approach to the extremely common idea of physical time, I will try to show it in a somewhat unexpected turn that might induce a revision of the traditional physics similar to that caused by the notion of relativity. This research is certainly not going to bring in any immediate industrial consequences, but, in the light of the rapid development of nanotechnologies, the structure of quantum time is attracting ever more attention, which stimulates the study of temporal correlation effects impossible without considering the hierarchy of time [7].

In our everyday life, to express time, we use standard time units, composing longer units from shorter ones. Thus, 60 seconds make a minute, 60 minutes make an hour, 24 hours make a day; we also use months counting from 28 to 31 days, years of 12 months, decades, centuries, milleniums. For some shorter time intervals we may introduce smaller units like milliseconds, microseconds… up to femtoseconds used in high energy physics. That is, the basic principle is to set up a number of counters, each capable of holding a number in a pre-defined range, and synchronize them so that, when some counter would reach its count limit, it would be reset with the next tick, and another (higher level) counter would be incremented. Counter reset means both setting the current count to zero and setting the new value of the count limit; in this way, we can remain flexible enough, to allow variable time units.

All we can evaluate is time intervals; there is no absolute time scale, and, to speak of a certain time moment, we have to refer to some arbitrary event taken for time zero, so that all the other events could be labeled with their time distance from the reference point. Negative times are introduced as “time before zero”.

In modern digital computers, timers are designed in a similar way, but there is less diversity in time units, and variable units are not allowed (though computer time can be converted into human time programmatically). Normally, in every computer there is a special microprocessor generating electric pulses in regular intervals, using a special battery when the computer is shut down. Different computers do not generally need to synchronize their timers, and any real time interaction between computers is usually governed by human time settings in the operating system. In distributed computing, when the same pool of data can be updated by different users from different locations, discrepancies in system time settings may cause trouble, and the newer operating systems have built-in synchronization mechanisms.

However, this habitual scheme does not always reflect the real situation. Indeed, it assumes that counters are operating in no time, and the next time interval begins immediately after the previous is over. Also, it is assumed that counters operate independently of any other processes, and of each other's operation. This is not so for real timers, and it cannot be so in principle, since any time measurement assumes interaction of some external object (e.g. the human observer) the with the counters, and this necessarily influences the whole counting processes, however infinitesimal this influence may be made. Counting speed depends on numerous factors, and no two timers can go in sync forever, which demands regular synchronization procedures, to maintain a common time for long enough. Historians know how difficult it is, to establish an exact dating of an event that occurred centuries ago, with many different calendar systems used since then, and different chronology used by different annalists. Similarly, in computers, timer operation often depends on all the rest of the system, and the system clock has to be adjusted from time to time, to allow any global synchronization.

One could admit that, on a large scale, no continuous time could be maintained at all, since there is no way to synchronize too distant processes and events. Similarly, for very short time intervals, we cannot control their relative phase, and no synchronization is possible. The centuries-old picture of time as a one-dimensional continuum seems to be a tribute to tradition, rather than an adequate approach to the problem. Of course, the principal inaccuracy of our time measurements does not mean that there is nothing unifying all such measurements, and the traditional vision of time may be enough for many applications. The idea of hierarchical time presented in this paper requires a more detailed investigation and search for special cases, to make it a sound methodology.

A positional system of calculation can serve as a simple model of digital time. We represent every real number with a sequence of digits chosen from the same discrete set $D = \{D_0, D_1, ..., D_{B-1}\}$:

$$n = \{d_N, d_{N-1}, ..., d_1, d_0\},$$

with $d_i \in D$. For instance, B can be set to 16 for the hexadecimal, or 10 for decimal, or 2 for binary system. For a fixed integer N, every sequence n corresponds to an integer number

$$\nu(n) = \sum_{i=0}^{N} k(d_i) B^i ,$$

and there are exactly K^{N+1} moments of time that can be labeled with the numbers ν. Obviously, the exact notation for the digits D_k does not play any role, provided we know the mapping $D_k \to k$. Since the integers ν are completely ordered, their enumeration from the minimal to maximal duration can be considered as a model of a *digital clock*, characterized with the cortege $C = \langle B, N, \tau \rangle$, where τ is the minimal measurable duration taken for the *time unit*; now, a pair $\langle C, t_0 \rangle$, with some initial time moment t_0 can be considered as a simple formal model of digital time.

In this model, there is a fixed hierarchical structure of counters, so that a higher-level counter, positioned closer to the left end of the row n, measures time intervals that are B times longer than those measured by the preceding lower-level counter. Once the clock has been started at time t_0, any event in some system can be associated with a certain moment of time ν; this association is called *time measurement*. Since real events do not need to depend on the clock operation (and, indeed, they should *not* interfere with it, for trustable clocking), the measurement procedure may be non-trivial, depending on the class of events registered. Generally, measuring time implies two stages:

1. An event is detected by the observer, who is believed to be able to categorize the observable situations and decide which situation corresponds to occurrence of the event, and which to its non-occurrence. In psychology, operation of that type is known as *binary discrimination*. An event always means some *state change*, transition from one class to another.

2. As soon as the event has been detected, the observer looks at the clock and reads the current row, thus determining the count ν, associating it with the occurrence of the event.

The operation inverse to time measurement will be referred to as *state measurement*, and it consists of the following stages:

1. The observer looks at the clock and reads the current row, thus determining the count ν.

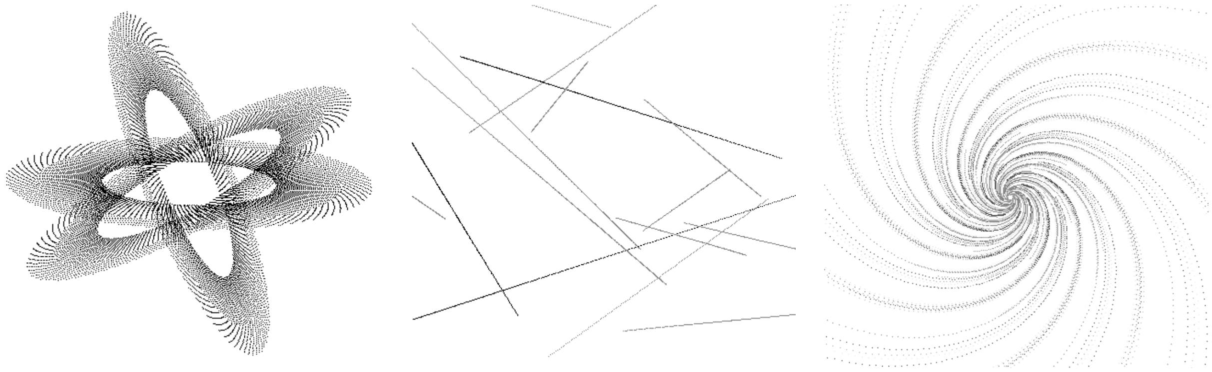
2. After that, the observer decides, to which class the current situation belongs, associating the state observed with the time moment $\nu\tau$.

Obviously, time measurement and state measurement do not coincide. Though time measurement can formally result in a series of the system's states associated with the moments of time, similar to that obtained via state measurement, the important difference is that, in time measurement, we always make a *retarded* association, in contrast to an *advance* association in state measurement. In a repeated observation, the observer alternatively looks either at the clock or at the system, and records the results as two parallel sequences:

$$\begin{tikzcd}
\nu_1 \arrow[dr] & & \nu_2 \arrow[dr] & & \nu_3 \arrow[dr] & & \nu_4 \arrow[dr] & & \nu_5 \arrow[dr] & \cdots \\
& s_1 \arrow[ur] & & s_2 \arrow[ur] & & s_3 \arrow[ur] & & s_4 \arrow[ur] & & s_5 \arrow[u] \; \cdots
\end{tikzcd}$$

In the best possible case, when switching from the system to the clock and back takes no time at all and $\nu_{k+1} = \nu_k + 1$, the observer can only conclude that event s_k has occurred at some time between ν_k and ν_{k+1}; inversely, the time of event s_k can only be measured up to the accuracy of the clock unit τ. However, if switching back and forth requires time comparable or greater than τ, the systematic error of time/state measurement may be more significant, fluctuating around some average value. Such clocking errors may be insignificant if the sampling rate (the time distance between successive measurements) is much greater than τ, so that the relative error introduced by finite measurement time is negligible. For instance, the observer may be registering some singular events (e.g. particle emission from a radioactive sample); if the particles are emitted once a million clock units, detection time of ten units does not much influence the accuracy of measurement. Nevertheless, one could observe the influence of time discretization in a long-term experiment, involving many sequences of events: the distribution of the results over time will not be smooth, manifesting an intricate pattern of gaps depending on the time/state measurement error as well as the clock calibration phase [8].

These effects are easily observed in computer modeling, where the essential discreteness of operation can produce quite unexpected patterns that do not always correspond to any physical phenomena. However, since any physical experiment (and technological process) is also necessarily discretized, such computer experiments can illustrate the nonlinearity of the first type inherent in human activity [9] and hint to a possible non-traditional usage of the already existing technologies. Inversely, the natural phenomena that seem to be quite different may be manifestations of the same regularity, viewed under different conditions. For instance, a simple program displaying a track of a linear 3-dimensional oscillator (SPIRA for Windows, version 1.1 [10]), with a slight change of projection parameters, can produce entirely different patterns:



Though, in computer industry, there are standards on timer characteristics, one can observe that different computers have timers that tick in slightly different pace, and this difference can add up to significant discrepancies on a wider time scale. For two hierarchical timers $\{C\}$ and $\{C'\}$, one obtains two time labels

$$\nu(n) = \sum_{i=0}^{N} k(d_i) B^i \quad \text{and} \quad \nu'(n) = \sum_{i=0}^{N} k'(d'_i) B'^i$$

for each event, and there is a problem of translating one scale into another, which is analogous to comparing the clocks in different frames of reference, in special relativity. Practically, such a translation is made using a third timer, with the time unit much less than both τ and τ', so that the different course counts could be reduced to the same fine count for the same event. Obviously, such a reduction is only possible to a finite accuracy, and hence there is no preferable timer that could be used to calibrate all the other timers.

In physics, time is normally associated with the phase of some periodical process. To measure longer times, one has to correlate several periodical processes with different periods, so that the phase of a slower higher level oscillation could be used for the count of faster lower level oscillations. No physical process is preferable in that sense and all of them are used for clocking in appropriate situations. The only distinguished physical process is the evolution of the Universe as a whole, and the existence of a universal physical time is obviously related to the singularity of the Universe, provided this is a one-directional process, with no phase repeated. However, in real life (including physical experiment and industry) one can hardly use such universal time, and we can only derive it as a kind of uniformity in the numerous hierarchies of timers. This dependence of time measurement on the specific hierarchies of physical processes determines the observable effects, and the same pattern will be observed until a fixed clock hierarchy can be used. Thus, the constancy of the speed of light, as observed in numerous experiments may be due to the traditional usage of light for timing, its being implicitly taken for reference. If we could have another physical process, with a different (but constant) propagation speed, the standard relativistic formalism would be revised.

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