Musical Scale Hierarchy

L'hiérarchie des Échelles Musicales

The Roots of the Model

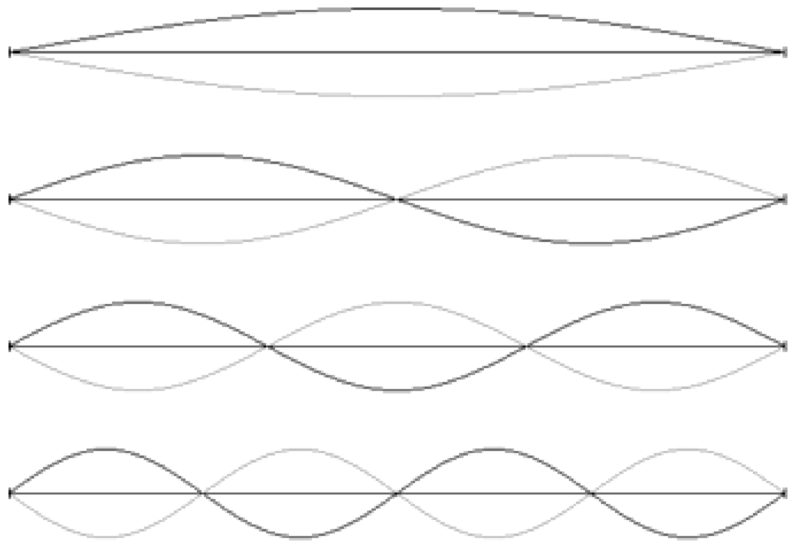
European musical tradition
Oriental music
Modern musical experiments
Physiology of pitch sensation
Physiology of perception
General psychology of activity
Quantum mechanics
Information theory
Computer-aided music composition
Experimenting with non-standard scales

Principal Milestones

1974-1976	Avdeev & Ivanov, Computer models of music composition
1977-1982	Avdeev, Mathematical models of interval perception
1979-1983	Koren, Hierarchical theory of human activity
1980-1984	Ivanov, General hierarchical approach
1983-1984	Avdeev, The completion of the mathematical model
1984-1991	Presentations in Russia
1990-1995	Major publications
1997	Death of Leonid Avdeev
2009	*The Genesis of the Musical Scales* (BODlib, St Petersburg, Russia) (written in 1992)

Pythagorean Tradition

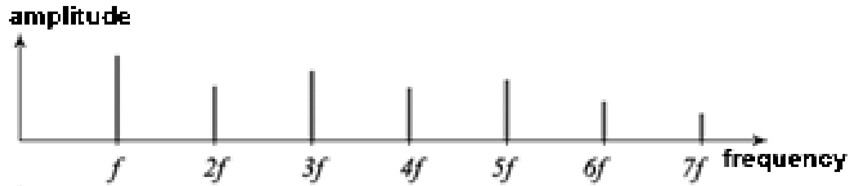
Pythagoras (VI century B.C.):
the natural scale,
the modes of string vibration



Musical intervals as ratios
of small integer numbers:

the octave	2 : 1
the fifth	3 : 2
the fourth	4 : 3
the major third	5 : 3
the minor third	6 : 5
the second	9 : 8
the seventh	7 : 4

Line spectrum:

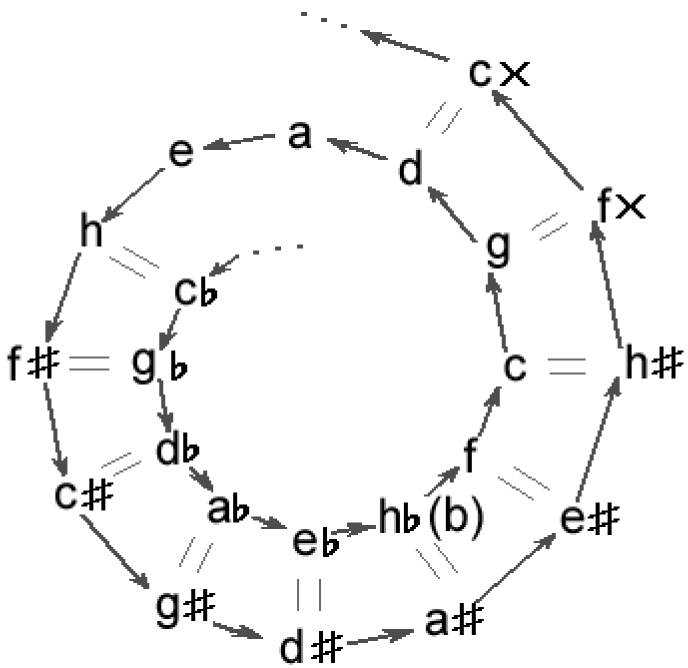


Pythagorean Scale

Iterating pure fifth → Pythagorean scale:

1	9 : 8	32 : 27	4 : 3	3 : 2	27 : 16	16 : 9	2
C	D	E	F	G	A	H	C

9 : 8   256 : 243   27 : 24   9 : 8   27 : 24   256 : 243   9 : 8



The circle of fifths

Identification of notes for every 12 steps

Pythagorean comma (74 : 73)
the exact value: 1.0136432

Mathematical Scales

Retaining the idea of pure tones for musical notes, but:

- ✓ Selection of other ratios for consonance
- ✓ Iterating other intervals
- ✓ Commatization of intervals
- ✓ Combinatory scales
- ✓ Artificial scales and modes

Two opposite views:

- Formal scales are mere approximations of the natural scale, and the natural intervals still determine the perception of music
- Modern music does not need definite pitch and any the sound effects are to replace the structured composition

A Few Problems

No exact pitch determination → no pure tones
$3/2 \approx 3001/2001$ → why talk about small integers?

The zone character of any aspect of musical perception
How are we to account for zones and predict their widths?

Human perception is hierarchically organized
How can we describe the hierarchy of musical structures?

Human perception evolves
How is this reflected in the history of music?

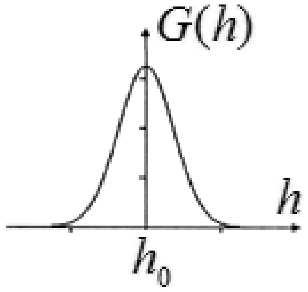
Traditional pitch systems of the different regions
Can they be explained in the same framework with the European music?

Modern music — is it physiologically justified?

Gauss Distribution

Nature does not know exact numbers

- no strict value perception
- statistical distributions
- perceptive sets



$$h = \log f$$

$$G(h-h_0;\sigma) = (2\pi)^{-1/2}\sigma^{-1}\exp\left[-\frac{1}{2}(h-h_0)^2 / \sigma^2\right]$$

Experimentally obtained yet by H. Helmholtz (XIX century)

Dispersion parameter:

- the accessible sharpness of perception (evolution)
- the level of perception (perceptional tuning)

Simple Tone Comparison

Acoustic timbre: musical instruments
Perception of pitch vs. perception of timbre

Inner timbre (a perceptional set):

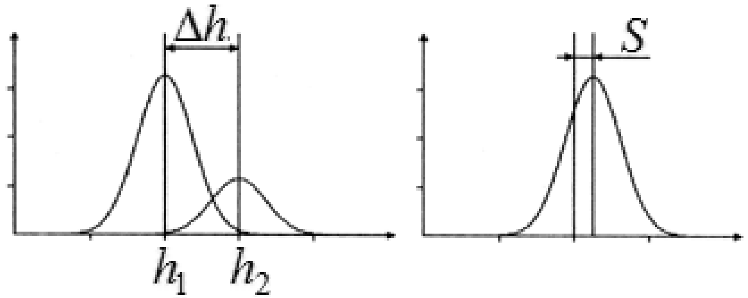
$\{t_n\}$, $n = 1 \ldots N$ — do not confuse with the acoustic timbre!

Quantum mechanical analogy: density operator

$$\Gamma(h) = \sum_{k=1}^{N} t_m G(h - h_k; \sigma)$$

Additivity hypothesis: pair wise comparison

Simple tone interference:



Representation by simple tones within the same set:

$$G_1' = (1 + \mu_{11})G_1 + \mu_{12}G_2 \rightarrow G(h - h_1 - S; \sigma)$$

$$G_2' = \mu_{21}G_1 + (1 + \mu_{22})G_2 \rightarrow G(h - h_2 + S; \sigma)$$

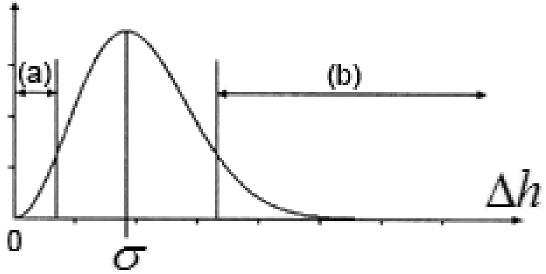
Discordance

Cross-entropy as a measure of interference:

$$H_c = \int \rho'(x) \ln\left(\frac{\rho'(x)}{\rho(x)}\right)$$

$$S = \mu \cdot \Delta h \cdot \exp\left(-\frac{1}{2}\Delta h^2/\sigma^2\right) + O(\mu^2)$$

$$H_c = \frac{1}{2} S^2/\sigma^2 = \frac{1}{2}\mu^2 (S^2/\sigma^2) \exp\left(- S^2/\sigma^2\right) + O(\mu^3)$$



Primary discordance:

$$d(\Delta h) = (\Delta h^2/\sigma^2) \exp\left(-\Delta h^2/\sigma^2\right)$$

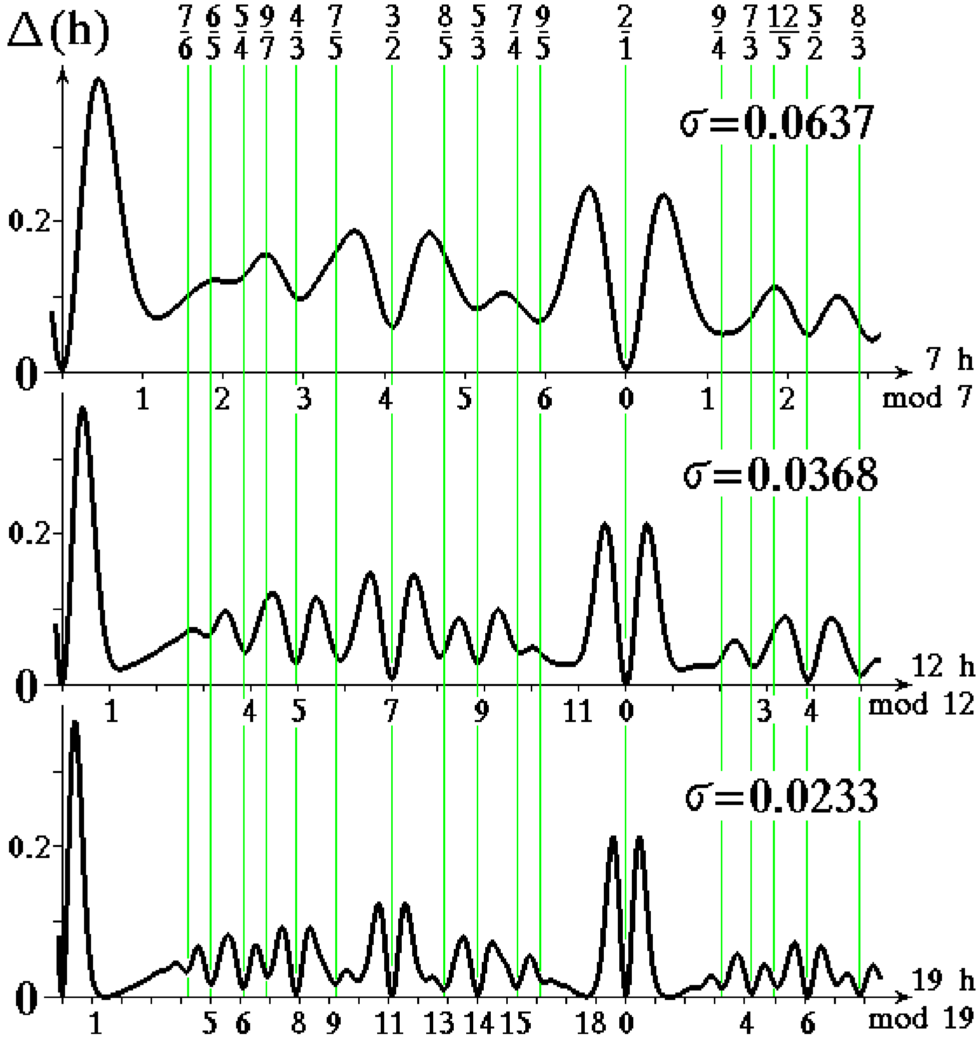
Symmetric in Δh → the both partials are equally informative

Zones of low interference: (a) and (b)
Essential interference (discord)

Discordance for Complex Tones

$$\Delta(h) = \sum_{m,n=1}^{N} t_m t_n d[h + \log_2(m/n)]$$

Minima at the positions approximately described by the ratios of small integers

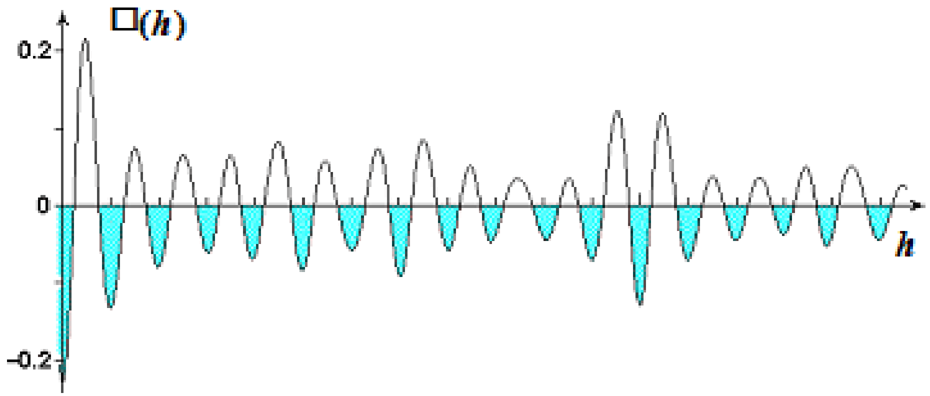


Dissonance

Determining zones:
deviations of discordance relative to the local average level

Dissonance function:

$$\square(h) = \Delta(h) - \int_{-\infty}^{+\infty} dx \Delta(x) G(x-h;\sigma)$$



A collection of zones of minimum dissonance can be a model of a musical scale

Perceptional Structure Formation

Inner timbre (a perceptional set): $\{t_n\}$, $n = 1 \ldots N$

How can we determine the partials t_n and their number?

Two criteria: stationarity and regularity.

Nonlinear signal processing →

only stationary patterns survive

Structure formation → the most articulated image stays

Regular discordance functions: Fourier transform

$$\widetilde{\Delta}(\nu) = \int_{-\infty}^{+\infty} dh \exp(-2\pi i \nu h) \Delta(h)$$

$$\Delta(h) = \int_{-\infty}^{+\infty} d\nu \exp(2\pi i h \nu) \widetilde{\Delta}(\nu).$$

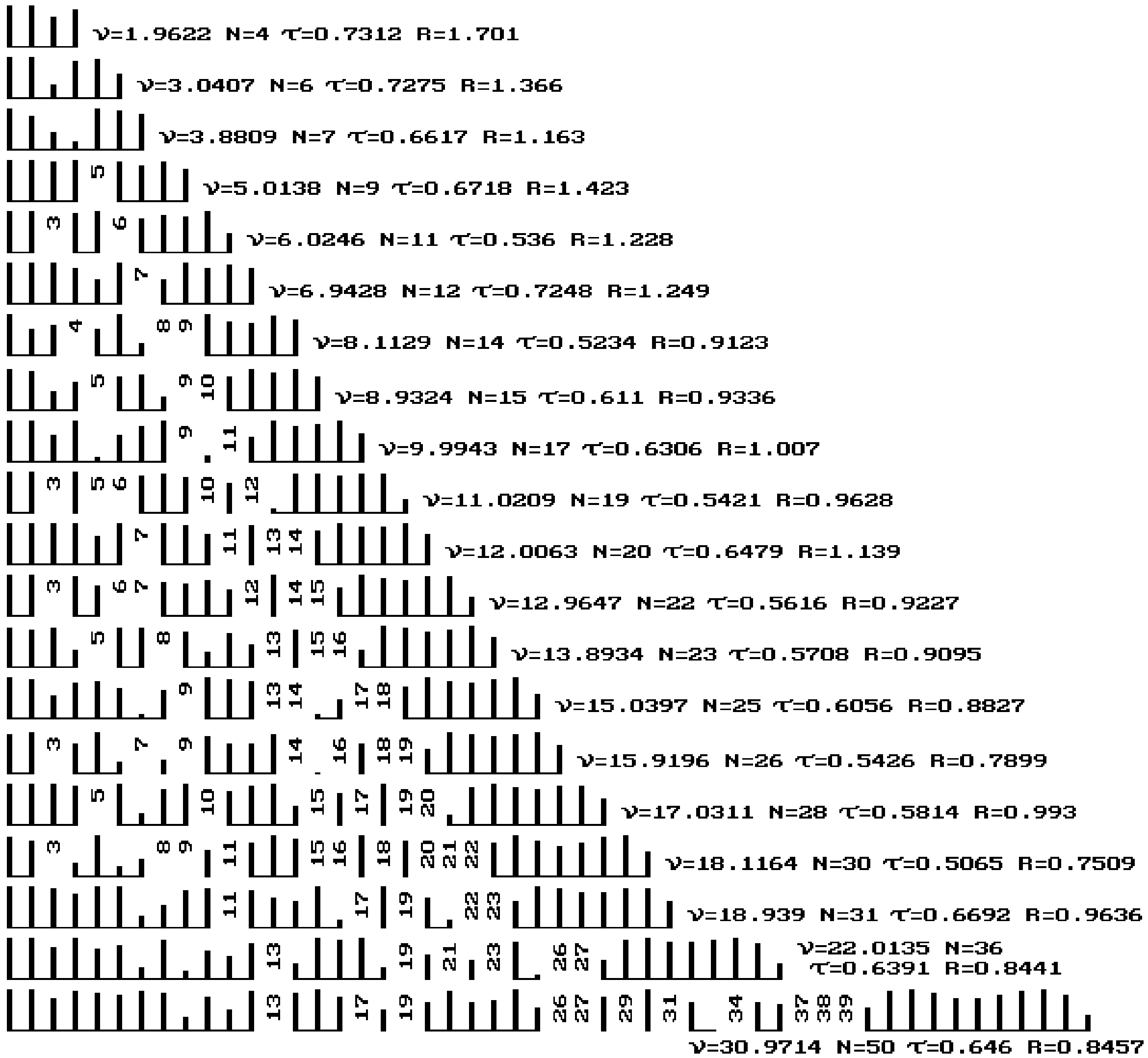
$$\widetilde{\Delta} = \widetilde{\Delta}_\sigma \widetilde{\Delta}_t$$

$$\widetilde{\Delta}_\sigma(\nu) = \sigma\sqrt{\pi}\left(1/2 - \pi^2 \sigma^2 \nu^2\right)$$

$$\widetilde{\Delta}_t(\nu) = \left| \sum_{n=1}^{N} t_n \exp(2\pi i \nu \log_2 n) \right|^2 .$$

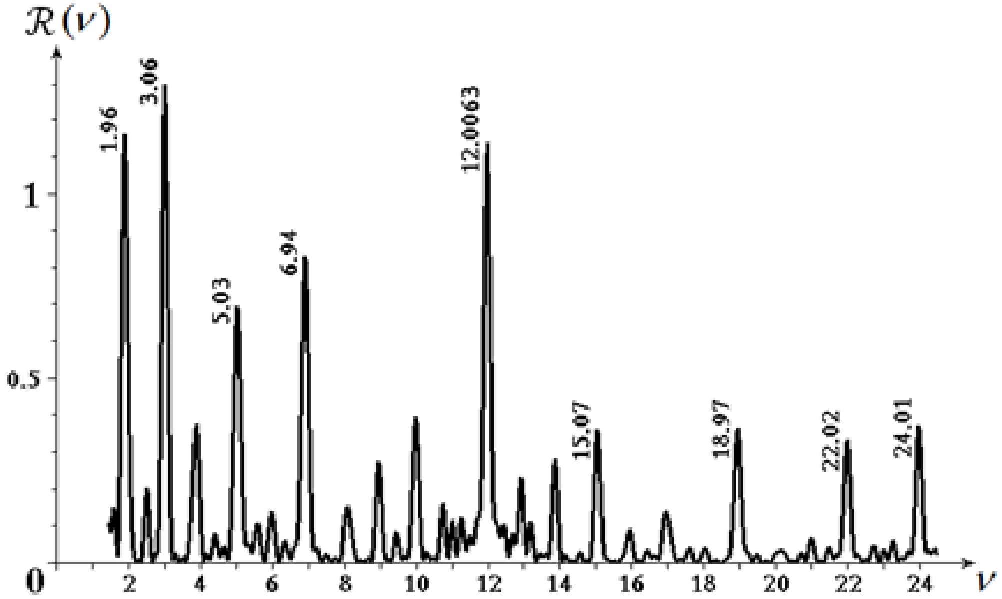
$$\mathcal{R}(\nu) = \widetilde{\Delta}_t(\nu) / \nu$$

Optimal timbres



Regularity and Substructures

The regularity function for the 12^{20} optimal timbre:



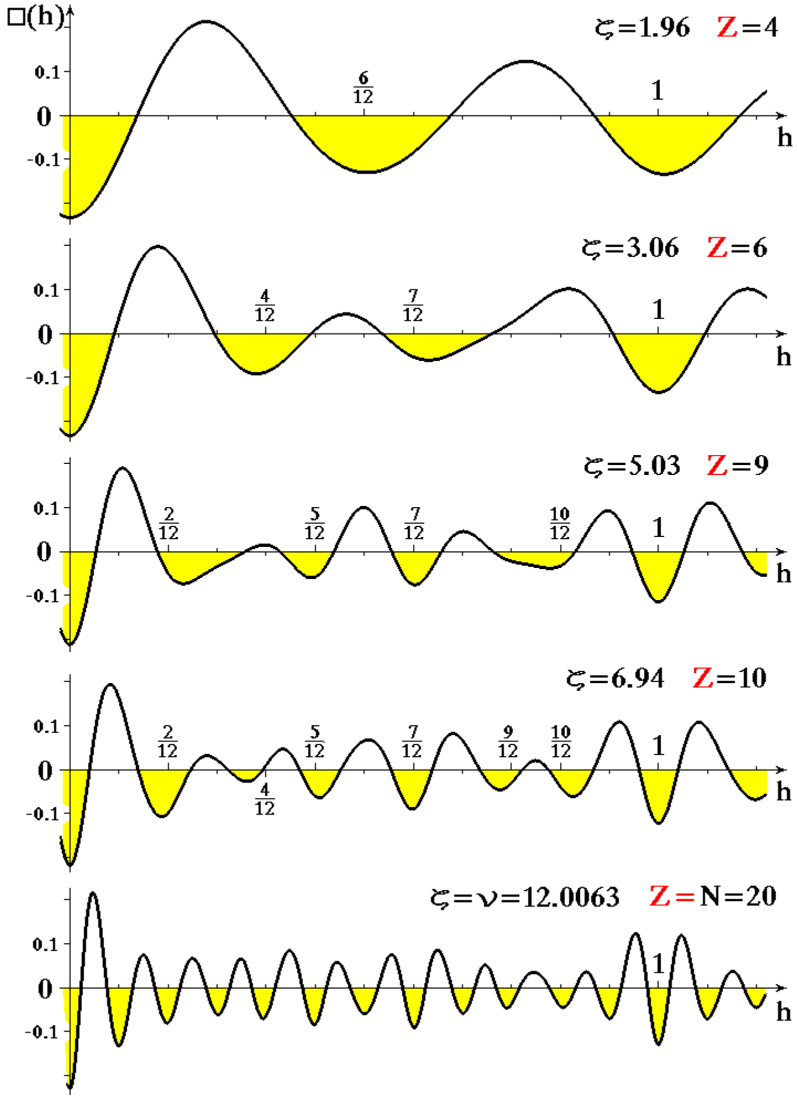
The points of maximum regularity → the possible subscales

The most pronounced peaks for $\nu \approx 2, 3, 5, 7, 12$

The standard structures of the standard 12-note scale:

an interval, a triad, pentatonic modes, diatonic modes

Embedded Scales in the 12-zone Scale



Inner Timbre Structure and Scale Quality

Formants:

harmonic	intervals used in harmony
modal	melodic intervals
modal/harmonic	melodic jumps
chromatic	alteration of intervals

Lability:

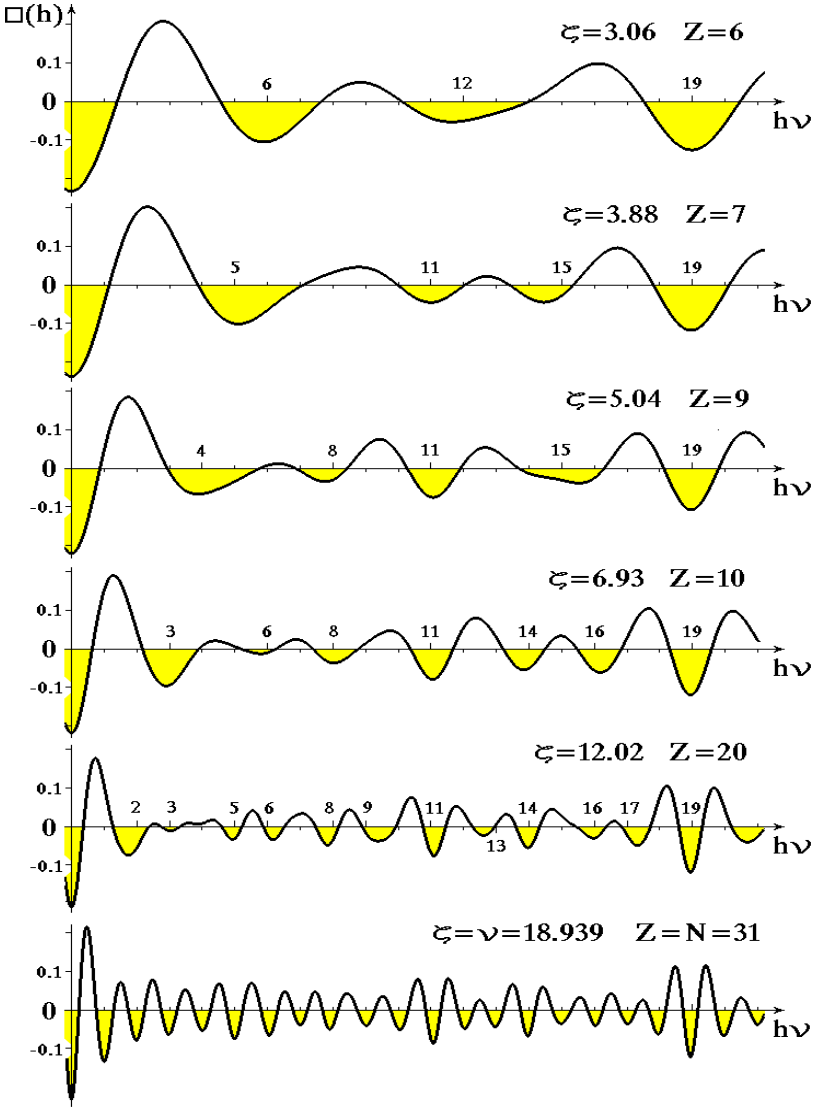
$$L = \log \frac{M(M-1) + 2MH}{H(H-1)}$$

Harmonic lability: any chord may play a centralizing role
Modal lability: weak melody centralization

Chromatic stability: alteration without function change

Anomalous, quintal, tertial, harmonic scales

Scale Hierarchy of the 19-zone Scale



19-tone music

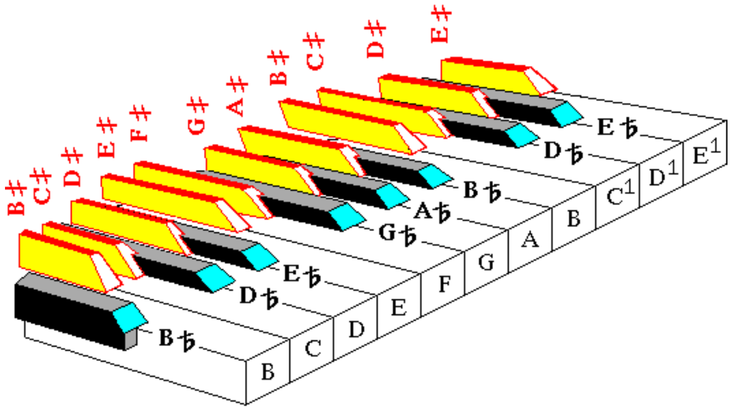
Stable harmonic scale

harmonic (10), modal (5), chromatically stable

the whole tone: 3 steps

the semitone: 2 step

the introductory semitone: 1 step

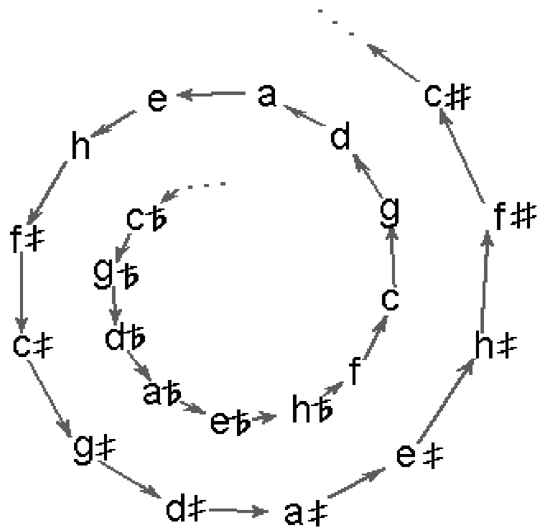


Principal embeddings:

19 ⊃ 7 ⊃ 3 (tonal music)

19 ⊃ 12 ⊃ 5 (hypertonal music)

Hypermodes:



Summary

- To explain the diversity of musical structures, a model of the psychological processes behind pitch perception is required
- The hierarchical nature of perception implies a similar hierarchy of the formal study
- The elements of quantum mechanics and information theory can be combined to predict the basic properties of pitch scales in music
- Each scale is associated with a hierarchy of zonal structures related to various aspects of music
- The functions of the admissible intervals can be described analyzing the structure of the inner representation of the scale (optimal timbre)

Summary

- A number of scales predicted by the model might serve as the basis for new creative experiments; in particular, the universal 19-zone scale has been described as qualitatively different from the traditional 12-tone system
- The set of theoretically obtained regular scales allows to reinterpret the history of European music and admits the inclusion of certain non-European pitch systems into the same cultural context
- The results obtained can be of interest from the general epistemological viewpoint as a practically useful example of scheme transfer from between natural sciences and humanities